Appendix 1: A Model with the Two Incentives

The Model Setup

In this section, I present a simple economic model to illustrate informed investors' behavior in the presence of a validation opportunity within networks. Consider a setting in which three types of risk-neutral investors trade claims to a risky asset. Type I investors are privately informed, while others are not. The privately informed investors receive a signal s that may be informative about the asset's terminal value $\tilde{v}$ with some probability. Specifically, the signal s is defined as:

$$s = \begin{cases} \tilde{v} & \text{with probability } q \\ \tilde{\varepsilon} & \text{with probability } 1-q, \end{cases} \tag{6}$$

where both $\tilde{v}$ and $\tilde{\varepsilon}$ are normally distributed with mean 0 and variance σ^2. All random variables are independent of each other. With this information structure, q represents the quality of the private signal s, where higher values of q indicate a more informative signal.

A critical assumption of this model is that type I investors decide whether to share their signal with uniformed investors (type N) within networks prior to trading in $t = 1$. If they choose to share, I and N collaboratively process the signal s and determine whether it is informative about the asset's terminal value or merely noise. This collaborative information processing between I and U is referred to as *validation*. Thus, type I investors must decide between two options:

1. Trade based on the noisy signal s, maintaining full informational advantage, or
2. Validate the signal by collaborating with N, leading to more efficient trading while splitting economic rents with N.

After making their sharing decisions and forming posterior beliefs about $\tilde{v}$, type I and N

investors choose their demand for the risky asset to maximize the expectation of

$$d_i(P_2 - P_1) - \frac{c}{2}d_i^2, \tag{7}$$

where d_i represents the demand of type i investors for the risky asset, P_1 and P_2 are prices at periods 1 and 2, respectively, and the second term reflects costs related to holding a position, which guarantees that investors have finite demands (Fischer et al. [2024]). Thus, type I and N investors are short-term speculators, aiming to profit from price changes while managing the costs associated with their positions.

The third type of investors, out-of-network investors (O_t), are also uninformed but do not participate in the validation process. These investors enter the market in each period and hold their positions until the terminal period. Their objective is to maximize the expected payoff:

$$d_i(v - P_t) - \frac{c}{2}d_i^2. \tag{8}$$

where v is the asset's terminal value, and P_t is the price at the time they enter the market.

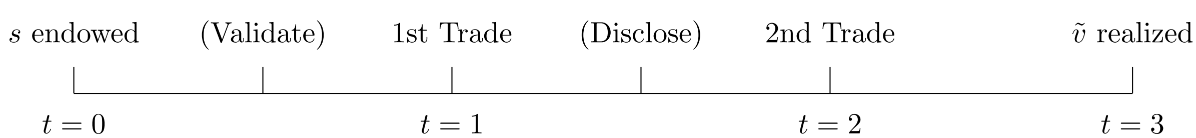
Additionally, noise traders are introduced into the model with aggregate demand in period t is given by $\frac{n_t}{c}$, where $n_t \sim N(0, \sigma_t^2)$, and is independent of all other random variables. The set of noise traders has measure 1, and there are π I type investors, ϕ N type investors, and ψ O_t type investors per noise trader. To simplify the analysis, I let $\psi \to \infty$ and derive risk-neutral prices. The cumulative holdings of all investors and noise traders are normalized to zero.

Before trading begins at $t = 2$, type I investors decide whether to publicly disclose their signals. This decision is separate from the validation process that occurs prior to $t = 1$, although what is disclosed depends on whether the signal was validated and the outcome of that validation. In summary, type I investors make two distinct information decisions that are not mutually exclusive: (1) sharing within networks prior to $t = 1$ and (2) public disclosure prior to $t = 2$.

These two choices involve different costs and benefits. Sharing within networks sacrifices some informational advantages but can improve the quality of private information. Furthermore, information leakage is limited within networks. In contrast, public disclosure prior to $t = 2$ does not result in the loss of informational advantages, as type I investors have already acted on the information. However, once disclosed, the signal gets reflected in the exit price, P_2, and all market participants have the same information.

The trading and information decisions of the investors characterize an equilibrium. For trading decisions, I focus on noisy rational expectations equilibria as in Grossman and Stiglitz [1980], where all investors are price takers, and prices reflect aggregated information, albeit imperfectly due to the presence of noise traders. For information decisions, the equilibrium condition requires that no out-of-network investors would alter their demand given their information set. The figure below illustrates the timeline of the model.

Figure: The Timeline of the Model



Note that s is not Gaussian but follows a mixture distribution consisting of v and ε. Assuming that both v and ε have zero mean and the same variance σ^2 simplifies the analysis considerably. The covariance between v and s is given by $Cov(v, s) = E[vs] - E[v]E[s] = E[vs] = q\sigma^2 + (1 - q)0 = q\sigma^2$, and the posterior expectation of $\tilde{v}$, given the realization of s, simplifies to:

$$E[\tilde{v}|s] = P(s = v|s) \cdot s = \frac{P(s = v)f_v(s)}{f_s(s)} \cdot s = \frac{q \cdot f_v(s)}{q \cdot f_v(s) + (1 - q) \cdot f_\varepsilon(s)} \cdot s = qs.$$

No Validation Equilibrium

I first characterize equilibrium conditions where type I investors do not share s within networks prior to $t = 1$. In period $t = 2$, both type I and N investors unwind their positions to exit the market, while type O_2 investors maximize equation (8) given their information set Ω:

$$d_{O_2} = \frac{E[\tilde{v}|\Omega] - P_2}{c}. \tag{9}$$

P_2 must satisfy the market clearing condition:

$$-\pi d_I - \phi d_N + \psi \left[\frac{E[\tilde{v}|\Omega] - P_2}{c} \right] + \frac{n_2}{c} = 0,$$
$$-\frac{c}{\psi} \left[\pi d_I + \phi d_N \right] + E[\tilde{v}|\Omega] + \frac{n_2}{\psi} = P_2,$$

and setting $\psi \to \infty$ yields $P_2 = E[\tilde{v}|\Omega]$.

In the following analysis, I focus on the case where type I investors disclose s publicly before $t = 2$. I will demonstrate that this disclosure is optimal for them, as shown in a later section. Assuming disclosure, the information set becomes $\Omega = \{s, P_1, P_2\}$, and the price at $t = 2$ is given by $P_2 = E[\tilde{v}|s, P_1, P_2] = E[\tilde{v}|s] = qs$.

In period $t = 1$, type I investors choose their demand to maximize equation (7) based on their private signal s:

$$d_I = \frac{E[P_2|s] - P_1}{c} = \frac{qs - P_1}{c},$$

while uninformed type N and O_1 investors' demands are

$$d_N = \frac{E[P_2|P_1] - P_1}{c} = \frac{q \cdot E[s|P_1] - P_1}{c}, \quad \text{and} \quad d_{O_1} = \frac{E[v|P_1] - P_1}{c}.$$

P_1 must satisfy the market clearing condition ($\pi d_I + \phi d_N + \psi d_{O_1} + \frac{n_1}{c} = 0$), which yields an equilibrium price of

$$P_1 = \frac{q}{\pi+\phi+\psi}\Big[\pi s + \phi E[s|P_1]\Big] + \frac{\psi}{\pi+\phi+\psi}E[v|P_1] + \frac{n_1}{\pi+\phi+\psi}.$$

Note that type N and O_1 investors can infer a sufficient statistic, $y_1 = q\pi s + n_1$, for P_1 with respect to s:

$$E[v|P_1] = qE[s|y_1] = \frac{q^2\pi\sigma^2}{q^2\pi^2\sigma^2 + \sigma_1^2} \cdot y_1,$$

and setting $\psi \to \infty$ yields

$$P_1 = E[v|P_1] = \frac{q^2\pi\sigma^2}{q^2\pi^2\sigma^2 + \sigma_1^2}\big(q\pi s + n_1\big). \tag{10}$$

Validation Equilibrium

Similarly, I characterize equilibrium conditions where type I investors share s within networks before $t = 1$. In period $t = 2$, both type I and N investors unwind their positions, and type O_2 investors' demand is given by equation (9).

If the validation outcome reveals that $s = v$, then in $t = 2$, the information set becomes $\Omega = \{v, P_1, P_2\}$ and P_2 becomes perfectly informative ($P_2 = v$). Conversely, if the validation reveals that $s = \varepsilon$, everyone knows that no one has been informed, and P_2 becomes uninformative ($P_2 = 0$).

In period $t = 1$, if $s = v$, then the demands for the risky asset are given by:

$$d_I = d_N = \frac{E[P_2|s=v] - P_1}{c} = \frac{v - P_1}{c}, \quad \text{and} \quad d_{O_1} = \frac{E[v|P_1] - P_1}{c},$$

and the price P_1 is determined as:

$$P_1 = \frac{\pi+\phi}{\pi+\phi+\psi} \cdot v + \frac{\psi}{\pi+\phi+\psi} \cdot E[v|P_1] + \frac{n_1}{\pi+\phi+\psi}.$$

From the price, uninformed investors can infer the statistic $y_1' = (\pi + \phi) \cdot v + n_1$, so we have:

$$E[v|y_1'] = \frac{(\pi + \phi)\sigma^2}{(\pi + \phi)^2\sigma^2 + \sigma_1^2} \cdot y_1',$$

and letting $\psi \to \infty$, we get:

$$P_1 = E[v|P_1] = \frac{(\pi + \phi)\sigma^2}{(\pi + \phi)^2\sigma^2 + \sigma_1^2} \cdot \big((\pi + \phi) \cdot v + n_1\big). \tag{11}$$

In contrast, if $s = \varepsilon$, then P_1 becomes uninformative, resulting in $P_1 = 0$. In summary, the optimal order flow for each type of investors is characterized in Lemma 1.

Lemma 1. *In the no-validation equilibrium, type I investors' demand in $t = 1$ is uniquely characterized by a function of the form*

$$d_I^\dagger = \frac{q}{c}\left[s - \frac{q\pi\sigma^2}{q^2\pi^2\sigma^2 + \sigma_1^2}(q\pi s + n_1)\right], \tag{12}$$

with all other types having zero demand. In the validation equilibrium, if $s = \varepsilon$, all types have zero demand; however, if $s = v$, both type I and N investors' demand in $t = 1$ is uniquely characterized by

$$d_I^\ddagger = d_N^\ddagger = \frac{1}{c}\left[v - \frac{(\pi + \phi)\sigma^2}{(\pi + \phi)^2\sigma^2 + \sigma_1^2}\big((\pi + \phi)v + n_1\big)\right]. \tag{13}$$

Information-sharing Decision

If type I investors do not share the signal s within their network prior to $t = 1$, then their ex-ante expected utility given s is

$$EU|_{\text{No Validation}} = E\left[d_I^\dagger(P_2^\dagger - P_1^\dagger) - \frac{c}{2}(d_I^\dagger)^2 \middle| s\right] = \frac{q^2\sigma_1^2}{2c} \cdot \frac{\pi^2 q^2\sigma^4 + s^2\sigma_1^2}{\big(\pi^2 q^2\sigma^2 + \sigma_1^2\big)^2}. \tag{14}$$

If type I investors share and validate the signal, then

$$EU|_{\text{Validation}} = E\left[d_I^{\ddagger}(P_2^{\ddagger} - P_1^{\ddagger}) - \frac{c}{2}(d_I^{\ddagger})^2 \middle| s\right] \quad = \frac{q\sigma_1^2}{2c} \cdot \frac{(\pi+\phi)^2\sigma^4 + \left(qs^2 + (1-q)\sigma^2\right)\sigma_1^2}{\left((\pi+\phi)^2\sigma^2 + \sigma_1^2\right)^2}. \tag{15}$$

Thus, type I investors are better off sharing the signal if and only if:

$$q \cdot \frac{\pi^2 q^2 \sigma^4 + s^2\sigma_1^2}{\left(\pi^2 q^2\sigma^2 + \sigma_1^2\right)^2} < \frac{(\pi+\phi)^2\sigma^4 + \left(qs^2 + (1-q)\sigma^2\right)\sigma_1^2}{\left((\pi+\phi)^2\sigma^2 + \sigma_1^2\right)^2}. \tag{16}$$

Note that the left-hand side of equation (16) always increases with q, while the right-hand side behaves more subtly:

$$\frac{\partial EU|_{\text{Validation}}}{\partial s^2} = \frac{(s^2 - \sigma^2)\sigma_1^2}{\left(\sigma^2(\pi+\phi)^2 + \sigma_1^2\right)^2}.$$

Thus, $EU|_{\text{Validation}}$ decreases with the quality of the signal q when the signal is not surprising ($s^2 < \sigma^2$) and increases with q when the signal is surprising ($s^2 > \sigma^2$). In contrast, $EU|_{\text{No Validation}}$ always increases with the signal quality q. Additionally, EU increases with s^2 in both cases, though $EU|_{\text{No Validation}}$ rises more quickly than $EU|_{\text{Validation}}$:

$$\frac{\partial EU|_{\text{No Validation}}}{\partial s^2} = \frac{q^2\sigma_1^2}{\left(q^2\pi^2\sigma^2 + \sigma_1^2\right)^2} > \frac{q^2\sigma_1^2}{\left((\pi^2+\phi^2)\sigma^2 + \sigma_1^2\right)^2} = \frac{\partial EU|_{\text{Validation}}}{\partial s^2}.$$

Consequently, the decision to share information depends on both the quality of the signal q and the extent to which the signal is surprising relative to σ^2, the variance of $\tilde{v}$. This information-sharing decision does not impact the order flow of uninformed investors, as the decision is not directional. Proposition 1 summarizes this relationship.

Proposition 1. *$q^*(s)$ and $s^*(\sigma^2)$ characterize a unique equilibrium for information-sharing within networks prior to $t = 1$, given the exogenous parameters. This equilibrium consists of the optimal information-sharing decision, Validation if (1) $|s| < s^*$ or (2) $q < q^*$ when*

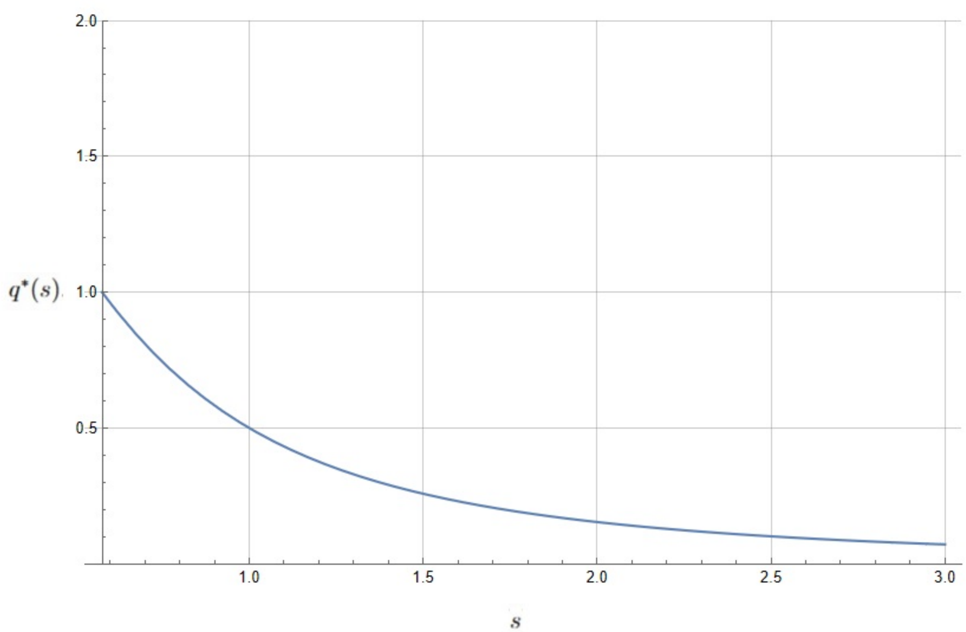
$|s| > s^*$, and No Validation otherwise, and the optimal demands characterized in Lemma 1.

In the following analysis, I assume $\pi = 0$, $\phi = 1$, and unit variances for simplicity. Setting $\pi = 0$ indicates that informed investors within networks are atomistic relative to other market participants, while $\phi = 1$ implies that there are as many uninformed in-network investors as noise traders. Assuming different weights π and ϕ leads to qualitatively similar results, varying only in the numerical thresholds for $q^*(s)$ and $s^*(\sigma^2)$. Under these simplifying assumptions, type I investors share their signal s if and only if:

$$\begin{cases} |s| < s^*(\sigma^2) = \dfrac{1}{\sqrt{3}}, & \text{or} \\ |s| > s^*(\sigma^2) \quad \text{and} \quad q < q^*(s) = \dfrac{2}{1+3s^2}. \end{cases}$$

The figure below illustrates $q^*(s)$ for the second condition. For the first condition, $q^*(s)$ is trivially equal to 1.

Figure: $q^*(s)$ for the second condition ($|s| > s^*$)



All else equal, a more surprising signal s offers a greater potential informational advantage for type I investors, so validation must reduce more uncertainty (i.e., require a lower q) to justify information sharing. This explains why $q^*(s)$ decreases as s deviates further from the zero mean.

Corollary 1. *In equilibrium, type I investors' incentive to validate decreases as s^2 increases, reflecting the greater potential informational advantage. To compensate for this advantage, $q^*(s)$ decreases, necessitating higher expected benefits from validation. Conversely, ceteris paribus, when q is low, type I investors are more likely to be incentivized to share their information within networks prior to trading in $t = 1$.*

Public Disclosure Decision

So far, I have assumed that public disclosure of s before $t = 2$ is optimal for type I investors. To formally demonstrate this, I compare the ex-ante expected utility across scenarios with and without disclosure, holding the validation condition constant. The ex-ante expected utilities with disclosure are given in equations (14) and (15). In the absence of disclosure, the second-period market price $P_2 = E[\tilde{v}|P_1] = P_1$ as investors trading in $t = 2$ are no better informed than those in $t = 1$. Thus, regardless of the validation condition,

$$EU|_{\text{No Disclosure}} = -\frac{c}{2}d_I^2,$$

and type I investors optimize it with $d_I = 0$. Since their order flow does not affect market prices, they have no incentive to trade if they cannot influence the exit price by publicly disclosing s. Furthermore, from equations (14) and (15), we see that $EU|_{\text{Disclosure}}$ is non-negative, irrespective of the validation condition. Thus, $EU|_{\text{Disclosure}}$ is weakly greater than $EU|_{\text{No Disclosure}}$, confirming the optimality of public disclosure of s before $t = 2$.

Lemma 2. *In any equilibrium, it is optimal for type I investors to disclose their signal s prior to trading in $t = 2$ in order to influence the exit price.*